Algebra Study Sheet

Logarithms and Log Properties

**Definition**
y = \log_b x \text{ is equivalent to } x = b^y

**Example**
\log_5 125 = 3 \text{ because } 5^3 = 125

**Logarithm Properties**
\log_b b = 1
\log_b 1 = 0
\log_b b^x = x
b^{\log_b x} = x
\log_b (x^r) = r \log_b x
\log_b (xy) = \log_b x + \log_b y
\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y

The domain of \log_b x \text{ is } x > 0

Special Logarithms
\ln x = \log_e x \quad \text{natural log}
\log x = \log_{10} x \quad \text{common log}
where \( e \approx 2.718281828\ldots \)

Conic sections

**Circle**
\( (x - h)^2 + (y - k)^2 = r^2 \)
Graph is a circle with radius \( r \) and center \((h, k)\).

**Ellipse**
\( \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \)
Graph is an ellipse with center \((h, k)\) with vertices \( a \) units right/left from the center and vertices \( b \) units up/down from the center.

**Hyperbola**
\( \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \)
Graph is a hyperbola that opens left and right, has a center at \((h, k)\), vertices \( a \) units left/right of center, and asymptotes that pass through center with slope \( \pm \frac{b}{a} \).

**Hyperbola**
\( \frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1 \)
Graph is a hyperbola that opens up and down, has a center at \((h, k)\), vertices \( b \) units up/down from the center, and asymptotes that pass through center with slope \( \pm \frac{b}{a} \).

Transformations of Functions

\( y = f(x) \pm c \) shifts the graph of \( f(x) \) up/down \( c \) units
\( y = f(x) \pm c \) shifts the graph of \( f(x) \) right/left \( c \) units
\( y = -f(x) \) reflects the graph of \( f(x) \) about the x-axis
\( y = f(-x) \) reflects the graph of \( f(x) \) about the y-axis
\( y = cf(x), c > 1 \) vertically stretches the graph of \( f(x) \)
\( y = cf(x), 0 < c < 1 \) vertically shrinks the graph of \( f(x) \)
\( y = f(cx), c > 1 \) horizontally stretches the graph of \( f(x) \)
\( y = f(cx), 0 < c < 1 \) horizontally shrinks the graph of \( f(x) \)
**Composite Functions**

\((f \circ g)(x) = f(g(x))\)

The composite function is obtained by replacing each occurrence of \(x\) in the equation for \(f\) with \(g(x)\)

**Difference Quotient**

\[
\frac{f(x + h) - f(x)}{h}, \quad h \neq 0
\]

**Rational Zero Theorem**

Possible rational zeros of a polynomial function = \(\frac{\text{Factors of the constant term}}{\text{Factors of the leading coefficient}}\)

**Binomial Theorem**

\((a + b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r\)